## How to Do Word Problems



## Study of Integers

In this chapter, we are are going to closely look at the number line system and study integers.


An integer is simply a number like $0,1,2,3$, and 4 , but unlike whole numbers, integers also include negative numbers like $-1,-2$, -3 and -4 . An integer cannot be a decimal or a fraction.

Consecutive integers are simply integers that follow each other by an increment of 1 , usually just one number after the other, like 1 , 2,3 and 4 or $-12,-11,-10$, and -9 .

Consecutive even or odd integers are simply integers that follow each other by an increment of 2 , like $0,2,4,6,8,10$ or $-11,-9,-7$, and -5 .

| Type | First | Second | Third |
| :---: | :---: | :---: | :---: |
| Consecutive Integers | $x$ | $x+1$ | $x+2$ |
| Consecutive Even Integers $^{1}$ | $x$ | $x+2$ | $x+4$ |
| Consecutive Odd Integers $^{2}$ | $x$ | $x+2$ | $x+4$ |

[^0]
## Example:

The sum of two consecutive integers is 231 . Find both integers.

## Solution:

Let $x$ and $x+1$ be the two consecutive integers.

$$
\begin{aligned}
\mid \text { First }+ \text { Second } & =231 & & \text { (Given Information) } \\
\hline x+\square x+1 & =231 & & \text { (Making Substitution) } \\
2 x+1 & =231 & & \text { (Simplify) } \\
2 x+1-1 & =231-1 & & \text { (Subtraction Property) } \\
2 x+0 & =230 & & \text { (Inverse \& Simplify) }
\end{aligned}
$$

## Solution(continued):

$$
\begin{aligned}
2 x=230 & \text { (Identity Property) } \\
x=115 & \text { (Division Property) }
\end{aligned}
$$

So the first integer is 115 , and for the next consecutive integer, we simply evaluate $x+1$ for $x=115$, that is $115+1=116$ inches.

The two consecutive integers are 115 and 116.

## Example:

Find two consecutive even integers such that twice the first one is 16 more than the second one.

## Solution:

Let $x$ be the first even integer and $x+2$ be the second consecutive even integer.

$$
\begin{array}{rlrl}
\hline 2 \cdot \text { First } & =\text { Second }+16 & & \text { (Given Information) } \\
\hline 2 \cdot x & =x+2 & x 6 & \\
\text { (Making Substitution) } \\
2 x & =x+18 & & \text { (Simplify) } \\
2 x-x & =x+18-x & & \text { (Subtraction Property) } \\
x & =18+0 & & \text { (Inverse \& Simplify) } \\
x & =18 & & \text { (Identity) }
\end{array}
$$

So the first even integer is 18 , and for the next consecutive even integer, we simply evaluate $x+2$ for $x=18$, that is $18+2=20$ inches.

The two consecutive even integers are 18 and 20.

## Example:

Find two consecutive odd integers such that the difference of three times first one and the second one is 100.

## Solution:

Let $x$ be the first odd integer and $x+2$ be the second consecutive odd integer.

$$
\begin{aligned}
\hline 3 \cdot \text { First }- \text { Second } & =101 & & \text { (Given Information) } \\
\hline 3 \cdot x-x-2 & =101 & & \text { (Making Substitution) } \\
3 x-(x+2) & =100 & & \text { (Use }(\ldots) \text { after }- \text { ) } \\
3 x-x-2 & =100 & & \text { (Distributive Property) } \\
2 x-2 & =100 & & \text { (Simplify) } \\
2 x-2+2 & =100+2 & & \text { (Addition Property) }
\end{aligned}
$$

## Solution(continued):

$$
\begin{aligned}
2 x+0 & =102 & & \text { (Inverse \& Simplify) } \\
2 x & =102 & & \text { (Identity) } \\
x & =51 & & \text { (Simplify) }
\end{aligned}
$$

So the first odd integer is 51 , and for the next consecutive odd integer, we simply evaluate $x+2$ for $x=51$, that is $51+2=53$ inches.

The two consecutive odd integers are 51 and 53.

## Example:

The length and the width a rectangular garden are two consecutive even integers. The perimeter of this garden is 298 feet. Find its dimensions.

## Solution:

Let $x$ be the measure of the width of this rectangle, therefore its length has to be $x+2$ since they are consecutive even integers.


$$
x+2
$$

Solution(continued):

$$
\begin{aligned}
P & =298 & & \text { (Given Information) } \\
2 L+2 W & =298 & & \text { (Perimeter Formula) } \\
2(x+2)+2 x & =298 & & \text { (Making Substitutior } \\
2 x+4+2 x & =298 & & \text { (Distributive Propert } \\
4 x+2 & =298 & & \text { (Simplify) } \\
4 x+2-2 & =298-2 & & \text { (Subtraction Propert } \\
4 x+0 & =296 & & \text { (Inverse \& Simplify) } \\
4 x & =296 & & \text { (Identity Property) } \\
x & =74 & & \text { (Division Property) }
\end{aligned}
$$

So the width is 74 feet, and for the length we evaluate $x+2$ for $x=74$, that is $74+2=76$ feet.

The dimensions of the garden are 74 ft . by 76 ft .

## Example:

Three sides a triangle are three consecutive odd integers. Find the measure of all three sides if its perimeter is 135 inches.

## Solution:

Let $x$ be the measure of the shortest side, therefore the other two sides are $x+2$ and $x+4$ since they are three consecutive odd integers.


$$
x+4
$$

## Solution(continued):

$$
\begin{aligned}
P & =135 & & \text { (Given Information) } \\
a+b+c & =135 & & \text { (Perimeter Formula) } \\
x+x+4+x+2 & =135 & & \text { (Making Substitution) } \\
3 x+6 & =135 & & \text { (Simplify) } \\
3 x+6-6 & =135-6 & & \text { (Subtraction Property) } \\
3 x+0 & =129 & & \text { (Inverse \& Simplify) } \\
3 x & =129 & & \text { (Identity Property) } \\
x & =43 & & \text { (Division Property) }
\end{aligned}
$$

So the shortest side is 43 inches, the next side is $43+2=45$ inches and the third side is $43+4=47$ inches.

The three sides of the triangle are 43,45 , and 47 inches.


[^0]:    ${ }^{1}$ The first integer $x$ must be an even integer.
    ${ }^{2}$ The first integer $x$ must be an odd integer.

